Sebastian Klein

Measure and Integration Theory
Exercise Sheet 1

8 September 2016

(will be discussed in the tutorial session on 13 or 15 September 2016)

1. The complement of axis-parallel cuboids.

Suppose $A, Q \in \mathfrak{Q}$ with $A \subset Q$. Show that $Q \setminus A$ is a union of finitely many, pairwise disjoint axis-parallel cuboids.

[Hint. A sketch (for the case $d = 2$) might be enlightening.]

2. About elementary sets. (The remainder of the proof of Proposition 1.6.)

Suppose $A, B \in \mathfrak{E}$.

(a) Show $A \setminus B \in \mathfrak{E}$.
[Hint. Represent A as disjoint union $\bigcup_{k=1}^{n} A_k$ of axis-parallel cuboids $A_k \in \mathfrak{Q}$ and use the equation $A \setminus B = \bigcup_{k=1}^{n} \big(A_k \setminus (B \cap A_k)\big)$.]

(b) Show $A \triangle B = (A \setminus B) \cup (B \setminus A) \in \mathfrak{E}$.

3. An alternative characterisation of elementary sets.

A set $A \subset \mathbb{R}^d$ is elementary if and only if it can be represented as the (not necessarily disjoint) union of finitely many axis-parallel cuboids.

[Hint. Proposition 1.6.]